Zitterbewegung, Acceleration, and Gravity

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This is a Machian analysis of gravity from the point of view of the zitterbewegung frequency. Zitterbewegung arises due to the fact that the only eigenvalues for the velocity operator of the Dirac equation are +c and -c. We use a model of gravity where gravitons stimulate matter to stimulate gravitons. We apply the model to constant acceleration and show that Einstein's coefficients for stimulated emission indicate that gravity is weak. We suggest a compatible preon model. We analyze the Schwarzschild metric and apply results on redshift to cosmology.

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Abstract

We propose that gravitons produce the force of gravity by stimulating matter to emit more gravitons in the same direction. When the velocity of an electron is measured, the only possible results (eigenvalues) are $\pm c$. A stationary electron must move back and forth at speed c resulting in what is called "Zitterbewegung" motion. This gives the instantaneous velocity of the electron in the velocity basis. Since gravity, over small distances, is equivalent to an acceleration, we compute the effect of an acceleration on the instantaneous velocity of the electron. We obtain exact equations for Einstein's coefficients for stimulated emission of gravitons. Looking for Feynman diagrams with the properties necessary to explain the coefficients, we show that the electron has to be composite and propose an old preon scheme with a composite interpretation of spin-1/2. We interpret black hole coordinate systems and apply these ideas to cosmology.

1 Zitterbewegung Acceleration

Soon after Dirac proposed the Dirac equation, Schroödinger found [1] that the only possible results for the measurement of an electron's velocity are $\pm c$. Thus the motion of an electron consists of a high frequency movement back and forth or "Zitterbewegung". [2, 3, 4] This essay addresses the interaction between zitterbewegung and uniform acceleration. We assume that gravitation is due to the stimulated emission of gravitons.

Let R, (L) denote the portion of time that an electron spends traveling at velocity $+1$, (-1). Then $R+L=1$, and using $c=1$, the velocity of the electron is:

$$\begin{aligned} v &= R-L, \\ \sqrt{1-v^2} &= \sqrt{(R+L)^2-(R-L)^2} = 2\sqrt{RL}. \end{aligned} \tag{1}$$

A relativistic particle in 1+1 dimensions, with a constant acceleration g follows a hyperbola:

$$x^2 - t^2 = 1/g^2. \tag{2}$$

Differentiating, and eliminating x and t gives:

$$\frac{d^2x}{dt^2} = g(1-v^2)^{3/2} = 8g(RL)^{3/2}. \tag{3}$$

We assume that gravitons are emitted when an electron transitions from R to L (and L to R), and that these gravitons act on another electron by stimulating similar transitions with further identical emissions. Corresponding to the two types of transitions, there are two types of gravitons, g_{RL} and g_{LR}.

Following Mach, we assume that the distant stars provide sufficient gravitons that essentially all transitions are stimulated; we ignore the spontaneous emission coefficients. Using $dx/dt = R - L$ we have:

$$\begin{array}{rcl} dR/dt &=& -I_{RL}B_{RL}R + I_{LR}B_{LR}L, \\ dL/dt &=& -I_{LR}B_{LR}L + I_{RL}B_{RL}R, \text{ so} \\ d^2x/dt^2 &=& -2I_{RL}B_{RL}R + 2I_{LR}B_{LR}L, \end{array} \tag{4}$$

where I_{RL}, (I_{LR}) is the flux of gravitons moving right, (left), and B_{RL}, B_{LR} are the Einstein coefficients. Eliminating d^2x/dt^2 from Eq. (3) and Eq. (4) gives:

$$\begin{array}{rcl} B_{RL} &=& 4(g/I_{RL})\sqrt{RL^3}, \\ B_{LR} &=& 4(g/I_{LR})\sqrt{LR^3}. \end{array} \tag{5}$$

And therefore:

$$\begin{array}{rcl} dR/dt &=& 4(g_{LR} - g_{RL})(RL)^{3/2}, \\ dL/dt &=& 4(g_{RL} - g_{LR})(RL)^{3/2}, \end{array} \tag{6}$$

where g has been split into its left and right moving parts $g = g_{LR} - g_{RL}$.

Equation (6) includes square roots of R and L. Define the vector ψ by

$$\begin{array}{rcl} \psi &=& \left(\begin{array}{c} \psi_R \\ \psi_L \end{array}\right) = \left(\begin{array}{c} \sqrt{R} \\ \sqrt{L} \end{array}\right), \\ 2\psi_R\psi_L &=& \sqrt{1-v^2}. \end{array} \tag{7}$$

Also see [4, 5]. We have

$$i\frac{d\psi}{dt} = i(g_{LR} - g_{RL})\left(\begin{array}{cc} 0 & (2\psi_R\psi_L)^2 \\ (2\psi_R\psi_L)^2 & 0 \end{array}\right)\psi. \tag{8}$$

The $\psi_L \to \psi_R$ transition rate is proportional to $\psi_L^3\psi_R^2$. Only even powers are naively expected since transition rates in quantum mechanics appear as the squared magnitudes of matrix elements. However, for a small addition m_{RL} to the background graviton matrix element M_{RL} we have:

$$(M_{RL} + m_{RL})^2 \approx M_{RL}^2 + 2M_{RL}m_{RL}, \tag{9}$$

so the net transition rate is linear in m_{RL} and the cube terms are okay, provided that the associated states are used three times in the matrix element. Thus ψ_L and ψ_R are composite with three components.

2 Prequarks

A simple preon model [6] has four types of prequarks $\{e_{L*}, e_{R*}, \bar{\nu}_{L*}, \bar{\nu}_{R*}\}$, each in three precolors $\{x, y, z\}$. The leptons are composed of three prequarks of the same type, while the quarks are mixed. The left handed fermions are:

$$\begin{array}{ll} e_L : & e_{Lx}e_{Ly}e_{Lz}, \\ \bar{u}_L : & e_{Lx}e_{Ly}\bar{\nu}_{Lz}, \quad e_{Lx}\bar{\nu}_{Ly}e_{Lz}, \quad \bar{\nu}_{Lx}e_{Ly}e_{Lz}, \\ d_L : & \bar{\nu}_{Lx}\bar{\nu}_{Ly}e_{Lz}, \quad \bar{\nu}_{Lx}e_{Ly}\bar{\nu}_{Lz}, \quad e_{Lx}\bar{\nu}_{Ly}\bar{\nu}_{Lz}, \\ \bar{\nu}_L : & \bar{\nu}_{Lx}\bar{\nu}_{Ly}\bar{\nu}_{Lz}, \end{array} \tag{10}$$

etc. This gives the fermions with their colors, weak hypercharge and weak isospin, but the implication is that the prequarks have spin 1/2. This is problematic in that a combination of three such objects decomposes as

$$2 \times 2 \times 2 = 4 + 2 + 2 \tag{11}$$

and one has an embarrassment of states, some with the wrong spin.

Recent research into the relationship between Feynman path integrals and the mutually unbiased bases of quantum information theory [7, 8] suggest a natural way to obtain spin-1/2 in the above prequark model. Spin-1/2 is assumed to be an emergent property that can be obtained by computing spin path integrals over mutually unbiased bases. One obtains three generations and spin-1/2 in the limit of long times. [9] These calculations are related to the Feynman checkerboard, [10] which in turn is a form of zitterbewegung. This also hints at the structure of the quark and lepton mixing matrices. [11]

The matrices in Eq. (8) are similar to a fermion mass term:

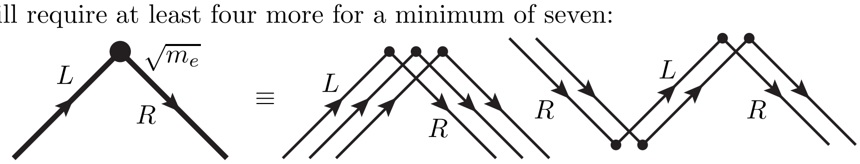
$$m_e(\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R) = \psi^\dagger \begin{pmatrix} 0 & m_e \\ m_e & 0 \end{pmatrix} \psi. \tag{12}$$

This suggests phenomenology with $\sqrt{m_e} = 2\psi_R\psi_L$. Koide's mass equations [12, 13, 14, 9] for the charged leptons m_g, and neutrinos $m_{\nu g}$ can be written as:

$$\begin{array}{rcl} \sqrt{m_{\nu g}} & = & 0.0990\sqrt{\text{eV}}\,[1 + \sqrt{2}\cos(2g\pi/3 + \pi/12 + 2/9)], \\ \sqrt{m_g} & = & 17716\sqrt{\text{eV}}\,[1 + \sqrt{2}\cos(2g\pi/3 + 2/9)], \end{array} \tag{13}$$

where g gives the generation. These also relate to spin path integrals. [9]

To achieve transition rates of $\psi_L^3\psi_R^2$ and $\psi_R^3\psi_L^2$ there must be three preons in ψ_L and ψ_R. Switching the three ψ_L preons to ψ_R will require at least three vertices. Only two ψ_R preons are involved and they have to return to ψ_R. This will require at least four more for a minimum of seven:



Any loops or other complications will add an even number of vertices giving $2n + 7$ as the power of the coupling constant. Thus the small value of the gravitational coupling constant could partly be explained as a result of a large number of vertices in its leading Feynman diagram.

3 Cosmology

A more complicated gravitational situation is the Schwarzschild metric. For this we must choose coordinates. QFT calculations will be simplified if we choose coordinates compatible with Dirac's gamma matrices on a flat background. Our desired coordinate system [15, 16] is Gullstrand-Painlevé (GP) coordinates:

$$d\tau^2 = (1 - 2M/r)dt^2 - \sqrt{8M/r}\, dt\, dr - dr^2 - r^2(d\theta^2 + \sin^2(\theta)d\phi^2). \qquad (14)$$

These are an example of coordinates implied by the Cambridge Geometry Group's "gauge theory gravity" (GTG). [17, 18, 16] This theory of gravity gives calculations identical to general relativity on every situation that can be described on a flat background metric.[1]

Putting $d\tau = d\theta = d\phi = 0$ and solving for dr/dt, we get the velocities of infalling and outgoing light:

$$dr/dt = -\sqrt{2M/r} \pm 1, \qquad (15)$$

where $c = 1$ is the speed of light far from the black hole. Any gravitation theory that assumes a flat background will have a variable speed of light. In GP coordinates, the variation is intuitive; infalling light is sped up by $\sqrt{2M/r}$, outgoing light is slowed down by the same amount. This suggests that one think of a black hole as a velocity field. The concept works beautifully not only for non rotating black holes, but also in the rotating and charged cases. [19, 20] The importance of velocity in our choice of coordinates mirrors our analysis of velocity in the paper's first section.

Schwarzschild coordinates have $d\tau/dt < 1$ outside the event horizon so all light is gravitationally redshifted. On the other hand, GP coordinates have $d\tau/dt = 1$ for particles with $dr/dt = -\sqrt{2M/r}$. Thus we interpret "gravitational redshift" as being entirely due to velocity relative to the velocity field. But over cosmological time, the graviton background increases and the zitterbewegung clock rate [2] of stationary particles increases.

In our model, the distant stars provide the gravitons that define the flat background that GTG is defined on. At cosmological time T, mass at distances up to c_gT can provide gravitons to us where $c_g > c$ is the speed of gravitons. Following a $1/r^2$ law, the number of gravitons we see per unit time is:

$$n(T) \propto \int_0^T 4\pi(c_gT)^2(1/c_gT)^2 dT \propto T, \qquad (16)$$

where T is a global clock unaffected by the graviton background. As our clock rates increase, the universe appears to grow in size. We can trade off changes in the size of the universe, the clock rate of particles, and the speed of light to give other descriptions. [21] And since energy is related to time, one can also assume a change in the particle masses. [22]

[1] Black holes not wormholes.

Let $\tau(T)$ be the length of one zitterbewegung defined clock period. We assume that τ scales inversely proportional to the background graviton density so we have:

$$\tau(T) = \tau_1 T_1 / T \tag{17}$$

where T_1 and τ_1 are the present cosmological time and zitterbewegung period. The number of zitterbewegung clock periods between $T = 0$ and time T is given by:

$$N(T) = \int_0^T (\tau(T))^{-1} dT = \frac{T^2}{2\tau_1 T_1}. \tag{18}$$

Since light moves over a path of length ΔX at constant speed $\Delta X/\Delta T = c$ in global coordinates, when we transform to the zitterbewegung clock N we scale the speed of light:

$$c(N) = \frac{\Delta X}{\Delta N} = \frac{\Delta X}{\Delta T}\sqrt{\frac{\tau_1 T_2}{2N}} = c_1 N^{-1/2}. \tag{19}$$

Since N corresponds to the usual time t, this is the same as the speed of light in Unzicker's new cosmology. [23]

Our calculations have been under the Machian assumption that the graviton background dominates the transition rate. Before then, the clock period would have been defined by spontaneous emission. Our assumption is that the Big Bang is due to the gravitational coupling constant. Since the constant is small, it follows that the time dominated by spontaneous emission was long. During that time, matter could have achieved thermal equilibrium effectively prior to the Big Bang.

In order to be compatible with GP coordinates, gravitons must travel at speeds in excess of c. The preon model (as well as Feynman's checkerboard in 3+1 dimensions) also implies speeds greater than c. Thus at the time before the combination of preons, the speeds at which matter moved must have exceeded c. Perhaps this is related to the puzzle of the angular correlations observed in the cosmic microwave background (CMB). [24]

An advantage of cosmology on a flat fixed background is that the observed flatness of the universe is automatic. In addition, in order to be consistent with solar system tests of gravity, gravitons have to interact with themselves to make more gravitons. [16] Thus the theory requires that $N(T)$ be nonlinear in T and dark energy is due to graviton / graviton interactions.

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